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A New High-Frequency Fluorescent Lamp Model

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as presented at IEEE Industry Applications Society Annual Meeting, St. Louis, Missouri, October 12-16, 1998

***Abstract* - A new model has been developed which describes the non-linear resistance of fluorescent lamps operating at high-frequency. A parabolic fit has been performed on actual lamp data which allows for simplified solutions to non-linear differential equations describing various fluorescent lamp output stages. A lamp waveform analysis has also been done using the model which shows an elliptical lamp current to be optimal.**

I. INTRODUCTION

Various models exist that describe the resistance of a fluorescent lamp operating at high-frequency [1]. These include both linear and cubic approximations. The linear, or resistor, approximation is given as,

$$v = iR \qquad \textbf{(1)}$$

where,

v = Instantaneous lamp voltage [Volts]
i = Instantaneous lamp current [Amperes]

This model is simple to use but has limited accuracy.

The cubic model is given as,

$$v = Ai + Bi^3 \qquad \textbf{(2)}$$

This is closer to the actual but more complicated to use. An approximation is needed which is both simple and accurate. The focus of this paper is on a simple and accurate parabolic model which allows for solutions to non-linear differential equations and provides a rapid method for obtaining ballast design parameters. This paper includes the parabolic model, optimum lamp waveform analysis, solutions to non-linear differential equations, and comparison with existing linear solutions.

II. THE PARABOLIC MODEL

Using a parabolic function to model the non-linear lamp resistance yields,

$$v = ki^2 \qquad \textbf{(3)}$$

Voltage and current measurements taken from a fluorescent lamp operating at high-frequency are plotted (Figure 1) together with the linear, cubic and parabolic models.

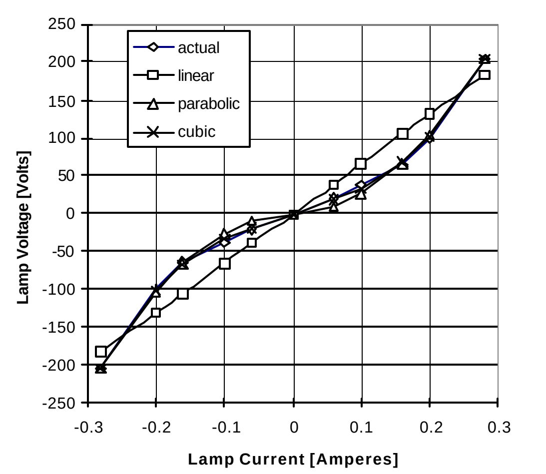

Figure 1, Non-linear lamp resistance and fitted models for T8/32W lamp type running at 50kHz (Plamp=30W, R=656 Ohms, k=2615, A=278, B=5740).

The parabola is a good fit for higher current regions of the v-i curve, where most of the power is consumed by the lamp. The cubic is linear for low values of current and does not predict the sharp increase in the actual voltage as the current increases. The parabola, however, increases sharper in voltage for a better fit at the higher current values and may predict higher harmonics. Overall, the parabolic is comparable to the cubic and easy to use.

III. The Optimum Lamp Waveform

The parabolic model was used to calculate various lamp parameters such as crest factor, peak voltage and current, voltage and current harmonics, and power, for the following lamp current waveforms:

$I = I_m$	(pulse)	**(4)**
$I = I_m \sin^2 wt$	(sine-squared)	**(5)**
$I = 4I_m ft$	(triangle)	**(6)**
$I = I_m \sin wt$	(sinewave)	**(7)**
$I = 2I_m\sqrt{ft}$	(parabola)	**(8)**
$I = I_m\sqrt{1-16f^2t^2}$	(ellipse)	**(9)**
$I = I_m$	(rectangle)	**(10)**

where,

$I_m =$ Lamp current amplitude [Amperes]

The waveforms were selected to transition in shape from a high, narrow pulse to a short, wide rectangle, in search of an optimum with which the lamp should be driven. Driving the lamp with a pure sinusoidal current, for example, gives a lamp voltage and power of the form,

$$V = ki^2 = kI_m^2 \sin^2 wt \quad \textbf{(11)}$$

$$P = \frac{4}{3p} kI_m^3 \quad \textbf{(12)}$$

When plotted (Figure 2), the lamp voltage has some zero-crossing distortion and increases sharply as the current increases yielding higher voltage harmonics. The current crest factor should be reduced to maximize lamp life and the maximum peak current should also be reduced to maximize ballast output stage efficiency. For these reasons, a sinusoidal lamp current is not necessarily the optimum.

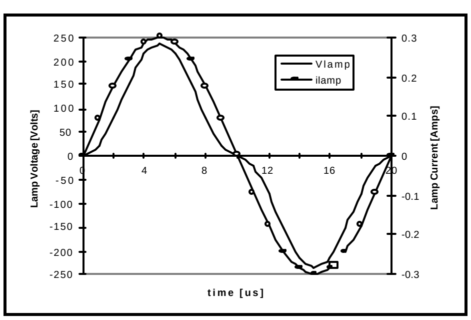

Figure 2, Sinewave lamp current and predicted lamp voltage for T8/32W lamp type running at 50kHz.

Driving the lamp with a triangular current gives a lamp voltage and power of the form,

$$V = ki^2 = 16kI_m^2 f^2 t^2 \quad \textbf{(13)}$$

$$P = \frac{1}{4} kI_m^3 \quad \textbf{(14)}$$

When plotted (Figure 3), peak currents and harmonics are higher than the sinusoidal case, and get worse moving in the direction towards a pulse.

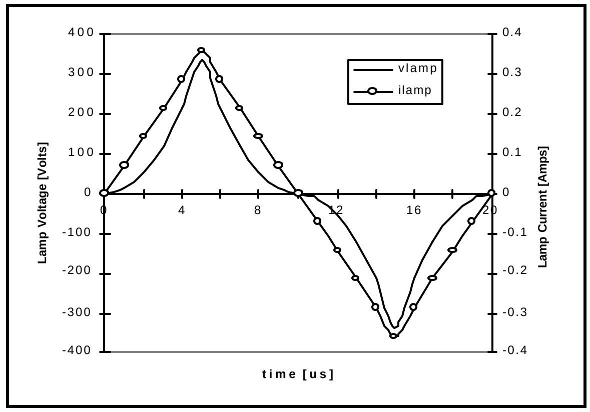

Figure 3, Triangular lamp current and predicted lamp voltage for
T8/32W lamp type running at 50kHz.

A rectangular current yields a crest factor of 1.0 and minimizes peak lamp current. The associated voltage and current harmonics, however, are high, tending to show that a squarewave is not an optimum either. In order to best select the optimum, the results from each waveform are normalized and plotted (Figure 4).

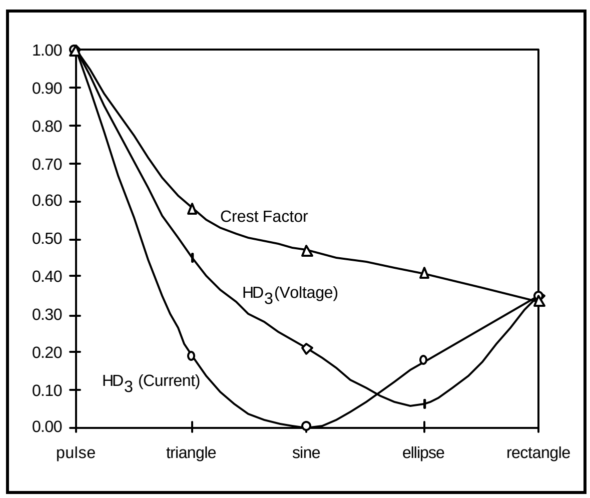

Figure 4, Optimum lamp waveform analysis summary for T8/32W lamp type.

The ellipse (Figure 5) proves to be more of an optimum waveform with an acceptable crest factor, lower peak currents and lower voltage harmonics due to a more sinusoidal lamp voltage.

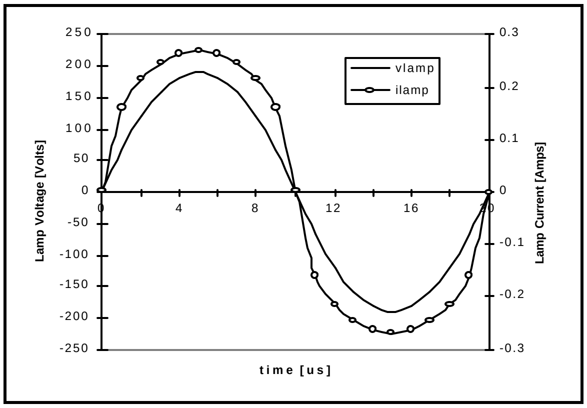

Figure 5, Elliptical lamp current and predicted lamp voltage for
T8/32W lamp type running at 50kHz.

IV. NON-LINEAR SOLUTIONS

The parabolic model allows for closed-form solutions of non-linear differential equations describing different fluorescent lamp output stages. A typical output stage consists of an L in series with the lamp (Figure 6).

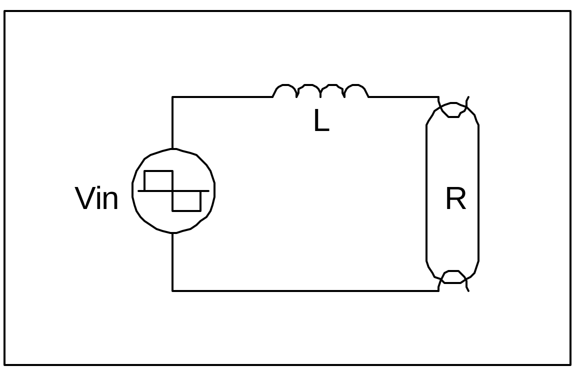

Figure 6, Series L-lamp output stage

Assuming the lamp has already been ignited and is running at nominal power, and using the model, the following non-linear differential equation results:

$$L\frac{di}{dt}+ki^2=V_{in} \qquad \textbf{(15)}$$

for which a solution exists of the form,

$$i(t)=\sqrt{\frac{V_{in}}{k}}\tanh\left(\frac{\sqrt{kV_{in}}}{L}t\right) \qquad \textbf{(16)}$$

The linear and non-linear solutions for the lamp current are plotted (Figure 7) over one-half of a complete cycle during steady state. From the plot it is seen that the non-linear solution rises sharper than the linear solution. This results in a slightly lower peak current for the non-linear case. A slight difference in phase can also be seen, which, for a given lamp power, will result in a higher operating frequency for the non-linear solution.

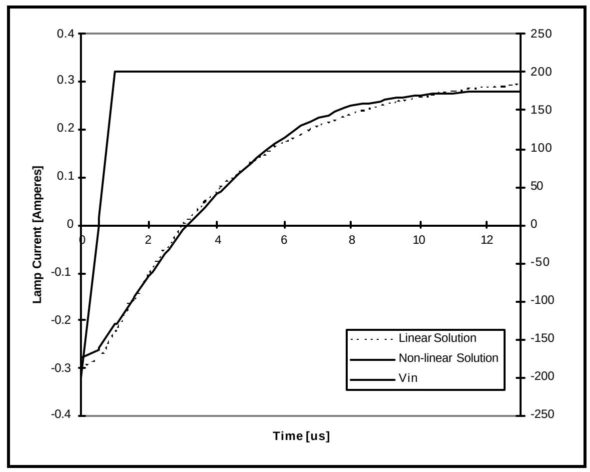

Figure 7, Linear and non-linear solutions for the series L-Lamp
output stage. Lamp type: T8/32W

The lamp voltage is also plotted (Figure 8), which shows a higher peak voltage and higher harmonics for the non-linear case due to the non-linear lamp resistance.

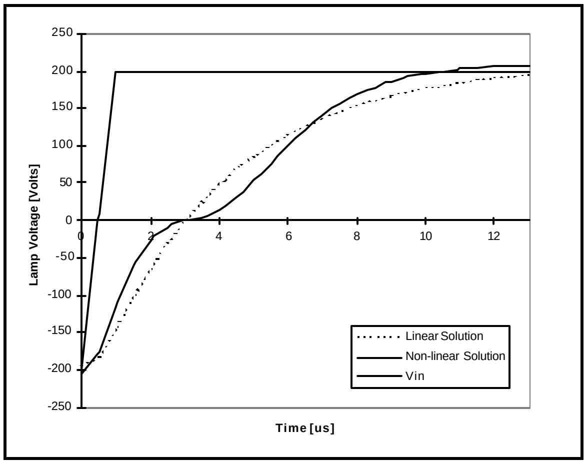

Figure 8, Linear and non-linear solutions for the series L-Lamp
output stage. Lamp type: T8/32W

Another popular output stage consists of an L-C-lamp series configuration (Figure 9).

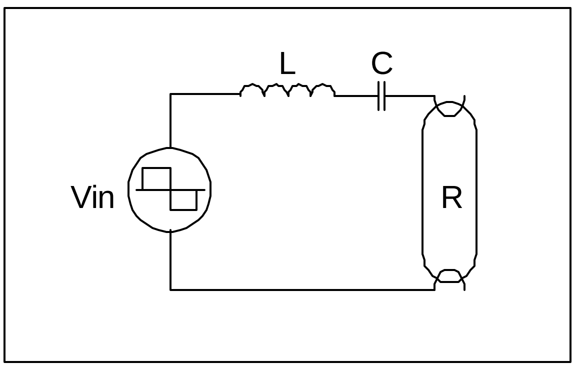

Figure 9, Series L-C-lamp output stage

If the cubic model is implemented, the result is a non-linear differential equation of the form,

$$L\frac{di}{dt} + Ai + Bi^3 + \frac{1}{c}\int idt = V_{in} \qquad \textbf{(17)}$$

The solution is not known for this Rayleigh/Van der Pol type equation. Using the parabolic, however, the following non-linear differential equation and solution results,

$$L\frac{di}{dt} + ki^2 + \frac{1}{c}\int idt = V_{in} \qquad \textbf{(18)}$$

$$Vin = L\frac{di}{dt} + ki^2 + \frac{L}{2kC}\ln\left[\frac{\frac{2kV_{in}C}{L}+1}{\left|2kC\frac{di}{dt}+1\right|}\right] \qquad \textbf{(19)}$$

or,

$$i^2 = \frac{1}{k}\left[-\frac{q}{C} + \left(V_{in} + \frac{L}{2kC}\right)\left(1 - e^{-\frac{2k}{L}q}\right)\right] \qquad \textbf{(20)}$$

where,

q = charge stored in capacitor C

These are new solutions and are not known to have been discovered before. Using **(18)**, **(20)** and basic circuit analysis, the inductor current and lamp voltage are again plotted (Figures 10 and 11) for the linear and non-linear solutions. From the plots, a slight difference in the peak current can still be seen, but the phase difference is now negligible. The lamp voltage solutions show even higher peak voltage differences and still higher harmonics in the non-linear case.

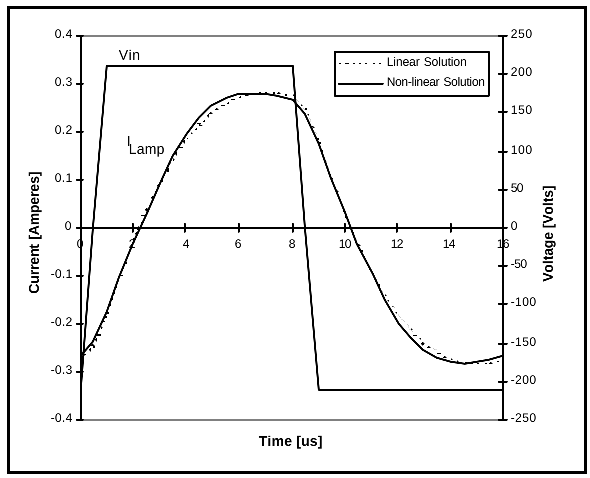

Figure 10, Linear and non-linear solutions for the Series L-C-lamp output stage. Lamp type: T8/32W

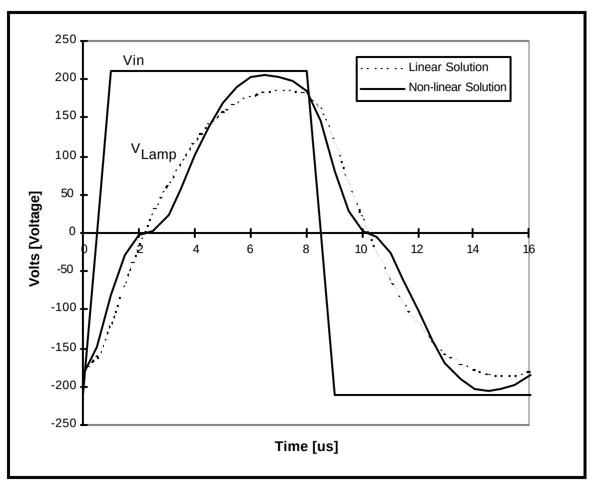

Figure 11, Linear and non-linear solutions for the Series L-C-lamp output stage. Lamp type: T8/32W

The traditional output stage consisting of an L in series with a parallel lamp and C (Figure 12) has also been solved.

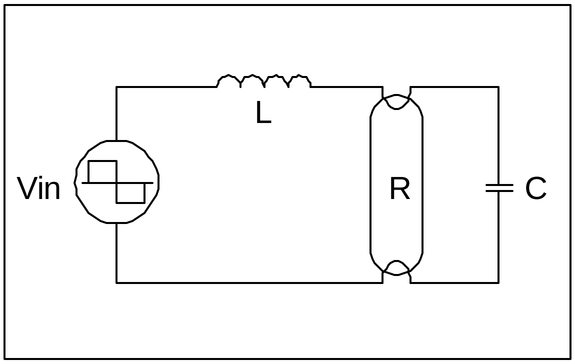

Figure 12, Series-L parallel lamp-C output stage

A mathematical solution exists which when plotted (Figures 13, 14 and 15) shows some interesting results. Here a “bump” occurs in the capacitor current (Figure 13) for the non-linear solution. This is due to the non-linear lamp resistance decreasing as the lamp current approaches zero. The decrease in lamp resistance gives an increase in the capacitor current and therefore a sharp increase in the lamp current near zero. This causes the non-linear lamp current to go to a lower peak than the linear solution, for a given lamp power. This results in a lamp current which appears elliptical in shape, giving rise to a more sinusoidal lamp voltage (Figure 15) as predicted earlier (Figure 5).

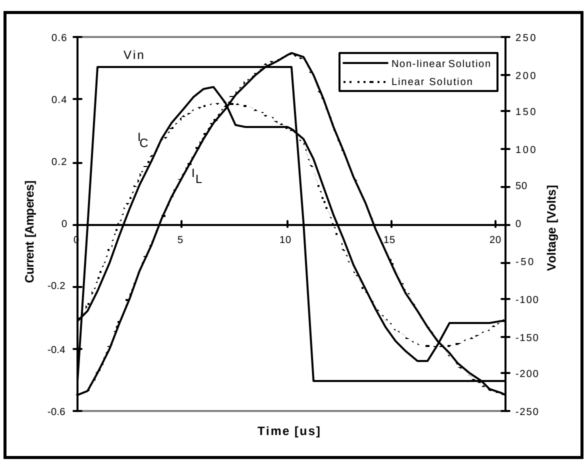

Figure 13, Linear and non-linear solutions for the series-L parallel

Lamp-C output stage. Lamp type: T8/32W

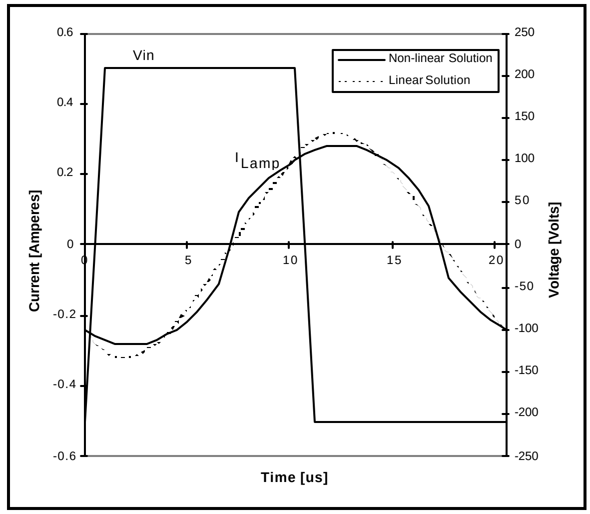

Figure 14, Linear and non-linear solutions for the series-L parallel

Lamp-C output stage. Lamp type: T8/32W

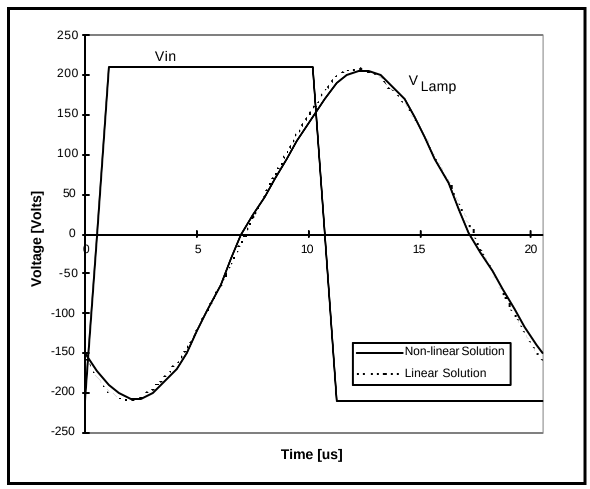

Figure 15, Linear and non-linear solutions for the series-L parallel

Lamp-C output stage. Lamp type: T8/32W

V. CONCLUSIONS

The parabolic model is simple, accurate and has greatly simplified lamp waveform analysis. The parabolic model has also provided new closed-form solutions to non-linear differential equations therefore reducing ballast design time. Future improvements include expanding the model to include dimming, in which *k* changes as a function of lamp power.

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